A Novel Stochastic Model for the Random Impact Series Method in Modal Testing

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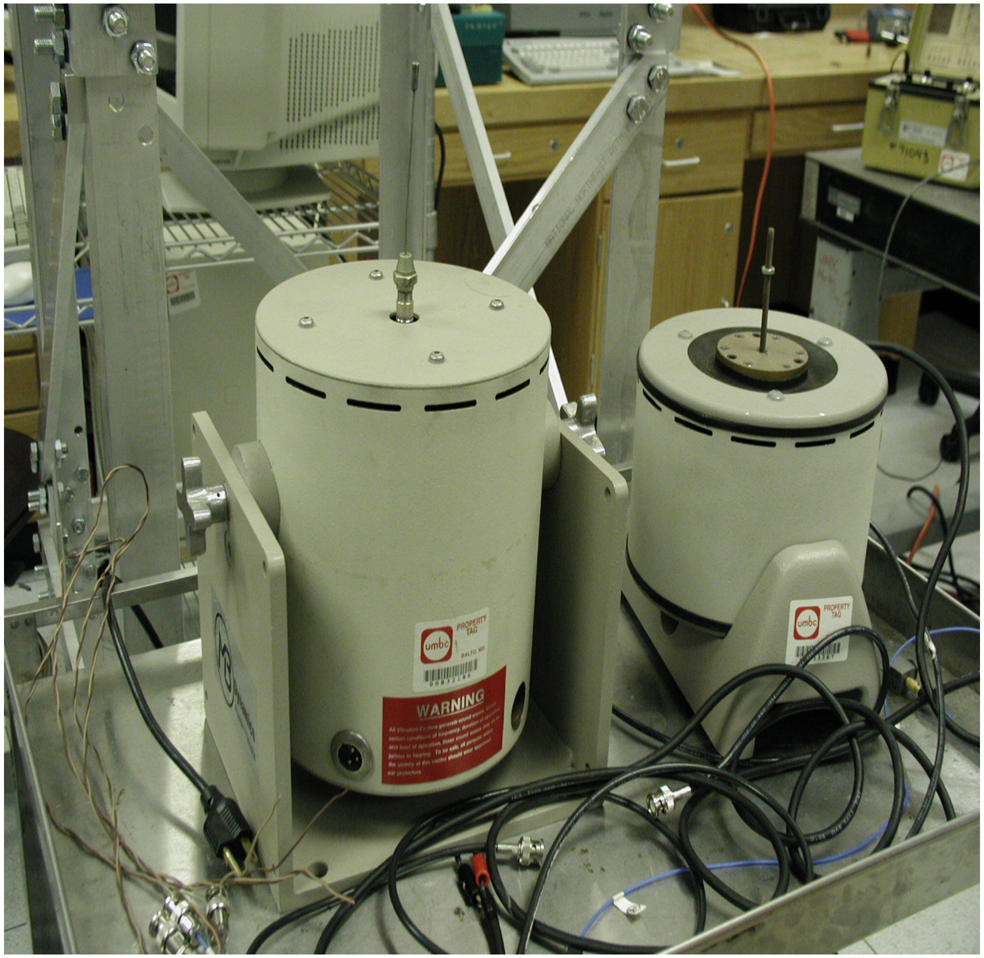
Shaker Test



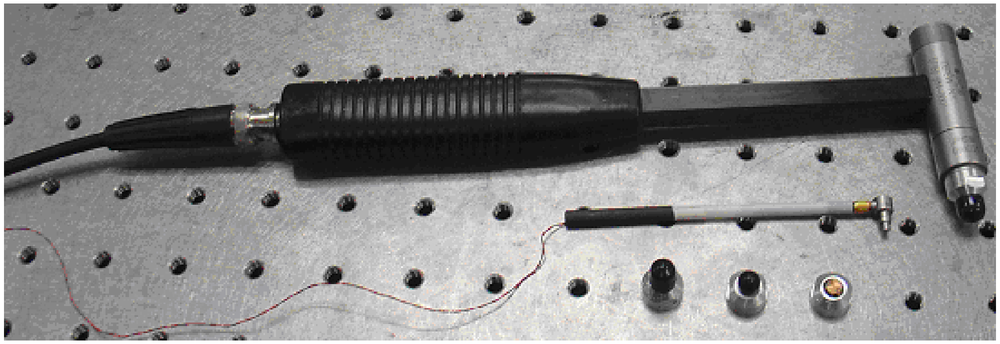
Advantages:

- Persistent excitation – large energy input and high signal-to-noise ratio
- Random excitation – can average out slight nonlinearities, such as those arising from opening and closing of cracks and loosening of bolted joints, that can exist in the structure and extract the linearized parameters

Disadvantages:

Inconvenient and expensive

Single-Impact Hammer Test



Advantages:

- Convenient
- Inexpensive
- Portable

Disadvantages:

- Low energy input
- Low signal-to-noise ratio
- No randomization of input – not good for nonlinear systems

Development of a Random Impact Test Method and a Random Impact Device

- **Combine the advantages of the two excitation methods**
  - Increased energy input to the structure
  - Randomized input – can average out slight nonlinearities that can exist in the structure and extract the linearized parameters
  - Convenient, inexpensive, and portable

- **Additional advantages**
  - A random impact device can be designed to excite very large structures
  - It cab be used to concentrate the input power in a desired frequency range

Novel Stochastic Models of a Random Impact Series

- **Random impact series (RIS) (Zhu, Zheng, and Wong, JVA, in press)**

  Random arrival times and pulse amplitudes, with the same deterministic pulse shape

- **Random impact series with a controlled spectrum (RISCS)**

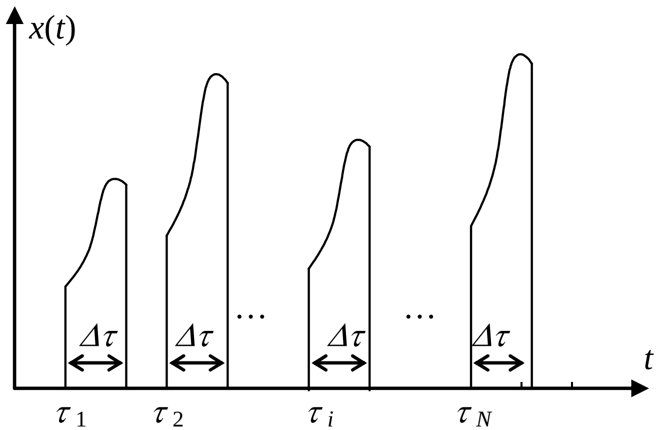
  Controlled arrival times and random pulse amplitudes, with the same deterministic pulse shape

Previous Work on the Random Impact Test

Huo and Zhang, Int. J. of Analytical and Exp. Modal Analysis, 1988

- Modeled the pulses as a half-sine wave, which is usually not the case in practice
- Analysis essentially deterministic in nature
  - The number of pulses in a time duration was modeled as a constant
  - No stochastic averages were determined
- The mean value of a sum involving products of pulse amplitudes were erroneously concluded to be zero

Mathematical model of an impact series



$$x(t) = \sum_{i=1}^{N} \psi_i y\left(t - \tau_i\right)$$

N - total number of force pulses

$y(\cdot)$ - shape function of all force pulses

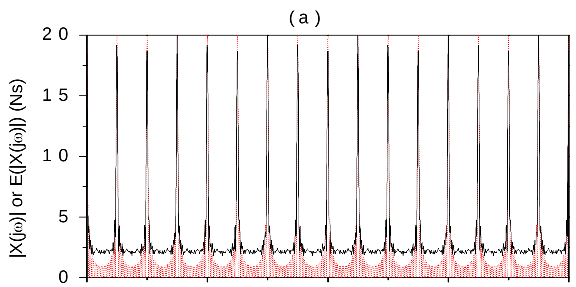
τ_i - arrival time of the *i*-th force pulse

ψ_i - amplitude of the *i*-th force pulse

$\Delta\tau$ - duration of all force pulses

The amplitude of the force spectrum for the impact series

- Deterministic arrive times with deterministic or random amplitudes

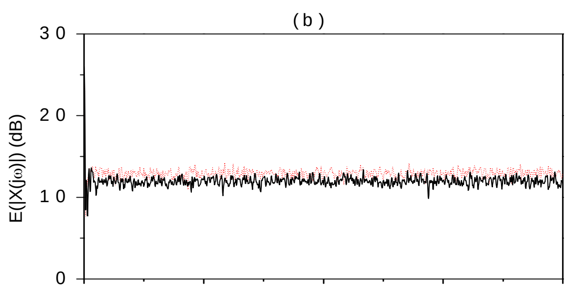


$\tau_i = i$ $\quad N = 20$

Solid line $\quad \psi_i = 1$

Dotted line $\quad \psi_i = normally\ distributed\ random\ variable$

- Random arrive times with deterministic or random amplitudes

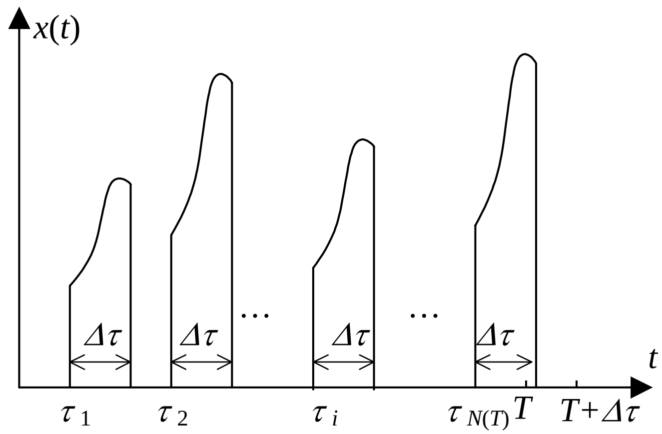


$\tau_i = normally\ distributed\ random\ variable\ around\ i$ $\quad N = 20$

Solid line $\quad \psi_i = 1$

Dotted line $\quad \psi_i = normally\ distributed\ random\ variable$

Time Function of the RIS*



$$x(t) = \sum_{i=1}^{N(T)} \psi_i y(t - \tau_i)$$

$$\tau_i \in (0, T] \;\; \text{and} \;\; t \in (0, T + \Delta\tau]$$

$N(T)$ - total number of force pulses that has arrived within the time interval $(0,T]$

$y(\cdot)$ - arbitrary deterministic shape function of all force pulses

τ_i - random arrival time of the i-th force pulse

ψ_i - random amplitude of the i-th force pulse

$\Delta\tau$ - duration of all force pulses

Challenge: A finite time random process with stationary and non-stationary parts

*Zhu et al., JVA, in press

Probability Density Function (PDF) of the Poisson Process N(T)

$$\mathrm{P}_{\{N\}}(n,T)=\frac{e^{-\lambda T}(\lambda T)^{n}}{n!}$$

$n=N(T)$ - number of arrived pulses

λ - constant arrival rate of the pulses

PDF of the Identically, Uniformly Distributed Arrival Times

$$p_{\tau_i}(\tau) = \begin{cases} \dfrac{1}{T} & 0 \le \tau \le T \\ 0 & \text{elsewhere} \end{cases}$$

where $i = 1, 2, \cdots, N(T)$

PDF of the Identically, Normally Distributed Pulse Amplitudes

(Used in numerical simulations)

$$p_{\psi_i}(\psi) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(\psi-\mu)^2}{2\sigma^2}} \qquad 0 < \psi < \infty$$

ψ - amplitude of the force pulses

μ - mean of ψ_i

σ^2 - variance of ψ_i

Mean function of *x*(*t*)

$$E\left[x(t)\right]=\begin{cases}\lambda E[\psi_1]W(t) & 0<t<\Delta\tau\\ \lambda E[\psi_1]W(\Delta\tau) & \Delta\tau\le t\le T\\ \lambda E[\psi_1]\left[W(\Delta\tau)-W(t-T)\right] & T<t\le T+\Delta\tau\end{cases}$$

where $W(t)=\int_0^t x(u)du$

$$E\left[x(t)\right]=\text{const} \qquad \text{when} \quad t\in[\Delta\tau,T]$$

Autocorrelation Function of *x*(*t*)

When $t \in [\Delta\tau, T]$

$$R_{xx}(k) = E\left[x(t_1)x(t_2)\right]$$

$$= \lambda E\left[\psi_1^2\right]\int_0^{\Delta\tau-|k|} y(u)\,y\left(u+|k|\right)du + \lambda^2 E^2[\psi_1]W^2(\Delta\tau),$$

where $k = t_2 - t_1$

x(*t*) is a wide-sense stationary random process in $[\Delta\tau, T]$

Average Power Densities of $x(t)$

- Comparison between $x(t)$ in $[0, T+\Delta\tau]$ and $[\Delta\tau, T]$

$t \in [0, T+\Delta\tau]$

$t \in [\Delta\tau, T]$

Non-stationary at the beginning and the end of the process

Wide sense stationary

Average power densities

$$S_1(\omega) = \frac{\left|X_{[0,T+\Delta\tau]}(j\omega)\right|^2}{T+\Delta\tau}$$

$$S_2(\omega) = \frac{\left|X_{[\Delta\tau,T]}(j\omega)\right|^2}{T-\Delta\tau}$$

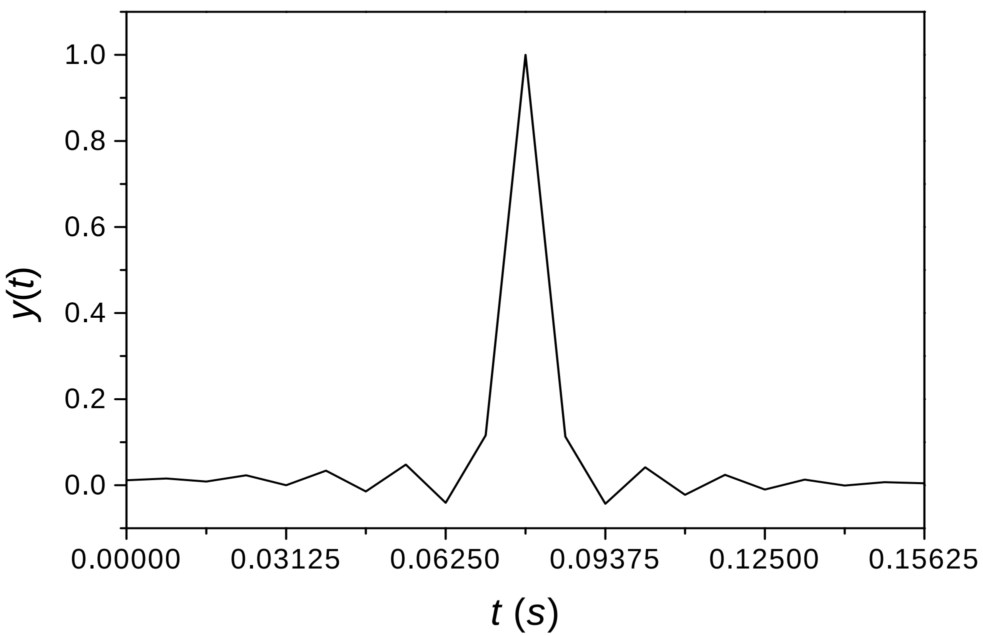
The expectations of average power densities

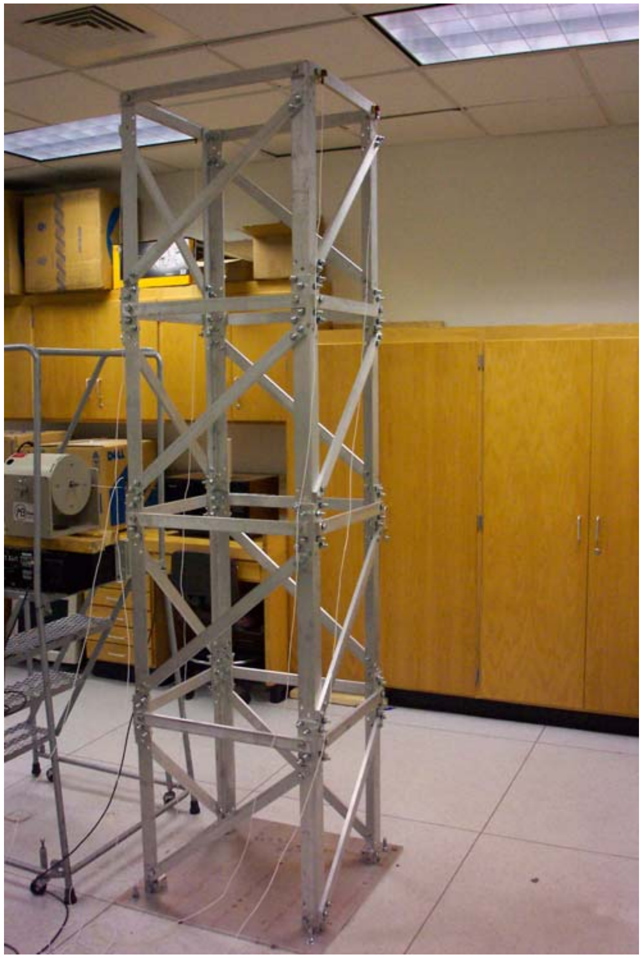
$$E[S_1(\omega)] = \frac{1}{T+\Delta\tau}\left[\int_{-\Delta\tau}^{\Delta\tau}\int_0^{\Delta\tau-|k|} x(v+|k|)x(v)\,dv\cos(\omega k)\,dk\right] \times \left\{2\lambda^2E^2[\psi_1]\frac{1-\cos(\omega T)}{\omega^2} + \lambda T E\left[\psi_1^2\right]\right\}$$

$$E[S_2(\omega)] = \lambda E\left[\psi_1^2\right]\int_{-\Delta\tau}^{\Delta\tau} dk \int_0^{\Delta\tau-|k|} x(u+|k|)x(u)\left(1-\frac{|k|}{T-\Delta\tau}\right)\cos\omega k\,du + 2\lambda^2E^2[\psi_1]W^2(\Delta\tau)\frac{1-\cos\omega(T-\Delta\tau)}{\omega^2(T-\Delta\tau)}$$

where $X(j\omega) = F[x(t)]$

Averaged, Normalized Shape Function

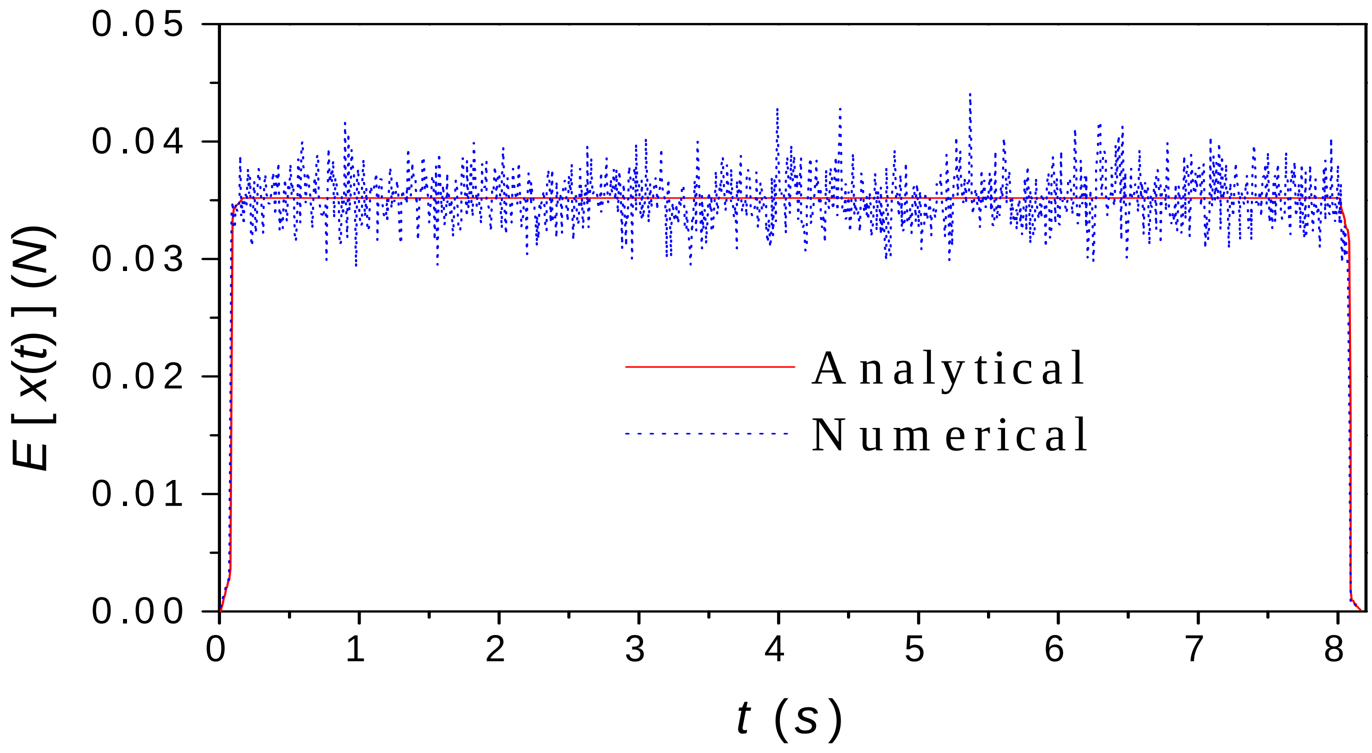




$$T = 8\ s \quad \Delta\tau = 0.15625\ s \quad \lambda = 4.14/\mathrm{s}$$

$$E[\psi_1^2] = 0.7163\,N^2 \qquad E[\psi_1] = 0.8239\,N$$

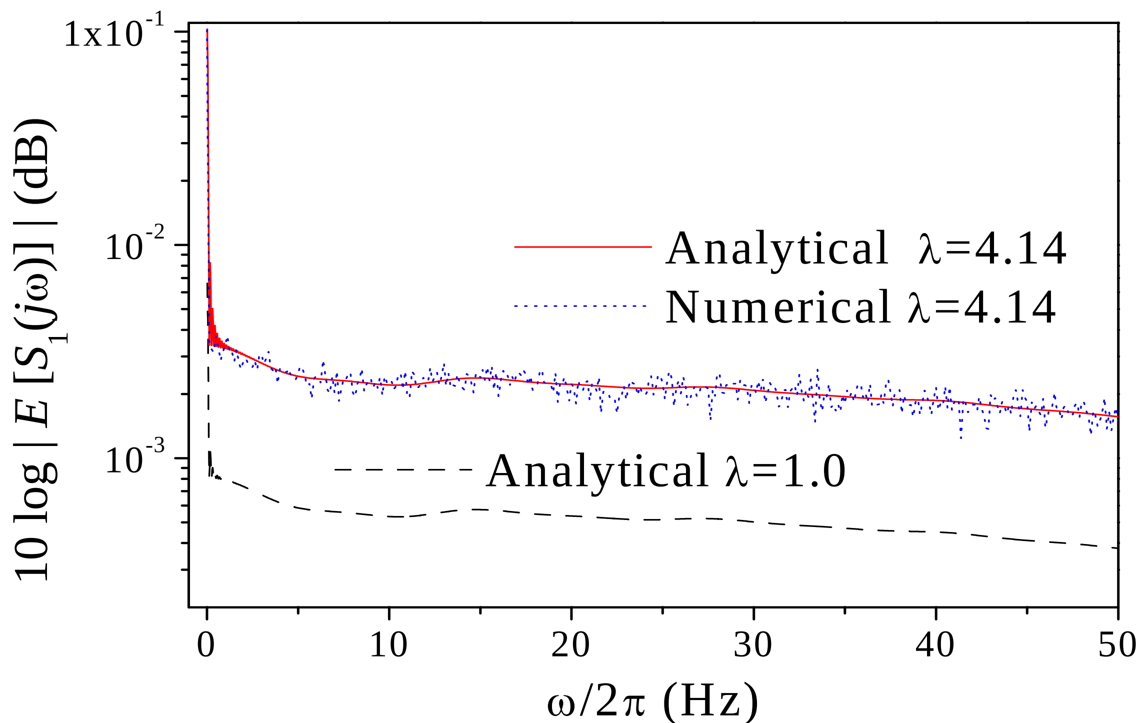
Comparison of Analytical and Numerical Results for Stochastic Averages



Mean function of $x(t)$

Comparison of Analytical and Numerical Results for Stochastic Averages (Cont.)

- Increasing the Pulse Arrival Rate Increases the Energy Input



Expectation of the average power density of $x(t)$ **in** $[0, T + \Delta\tau]$

A single degree of freedom system under single and random impact excitations

- Sampling time

  Ts=16 s

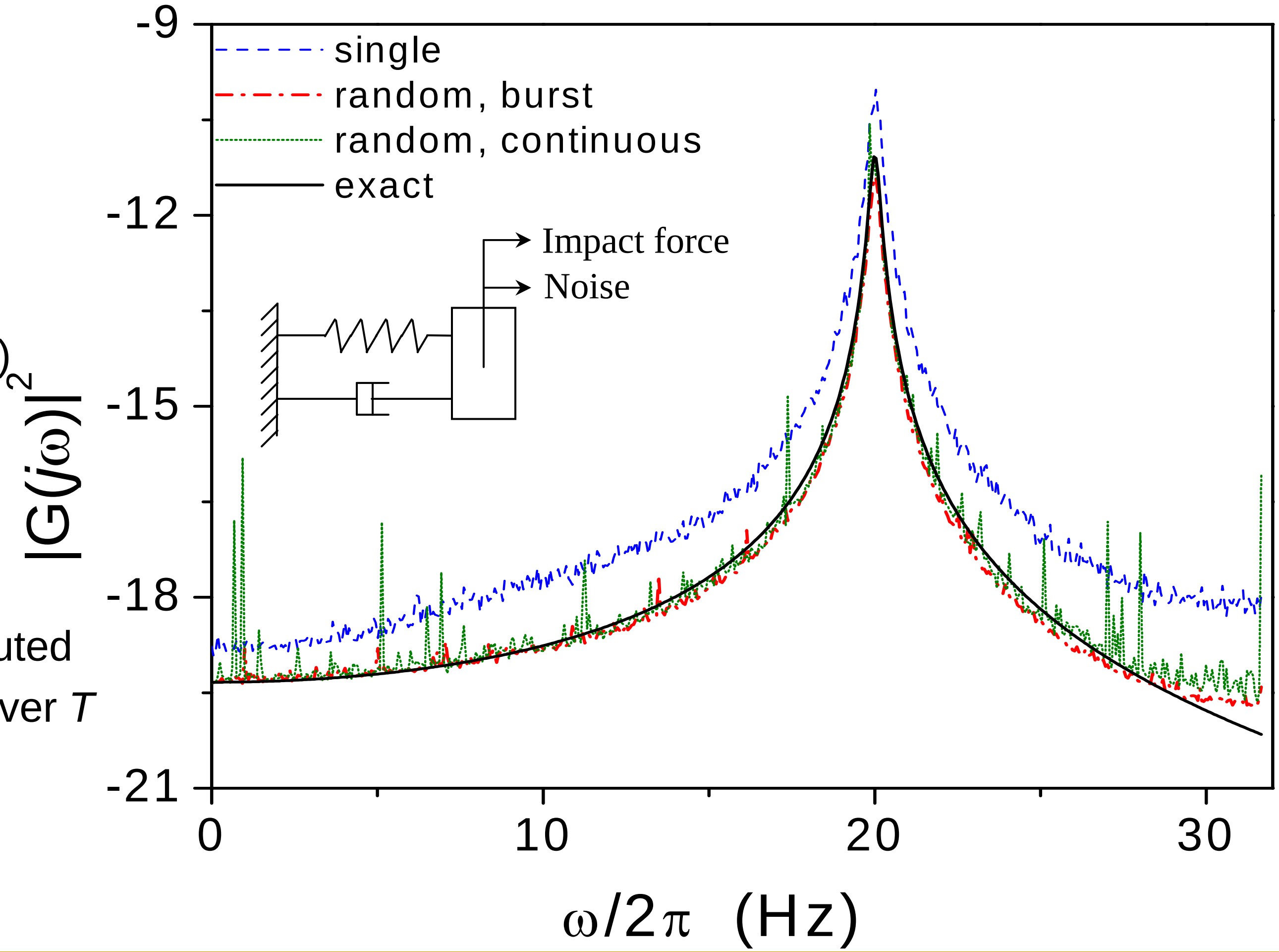
- Excitation time

  T=16 s (Continuous)

  T=11.39 s (Burst)

- Arrival times

  t_i =Uniformly distributed random variables over *T*



A single degree of freedom system under multiple impact excitations (cont.)

- Sampling time

  Ts=16 s

- Excitation time

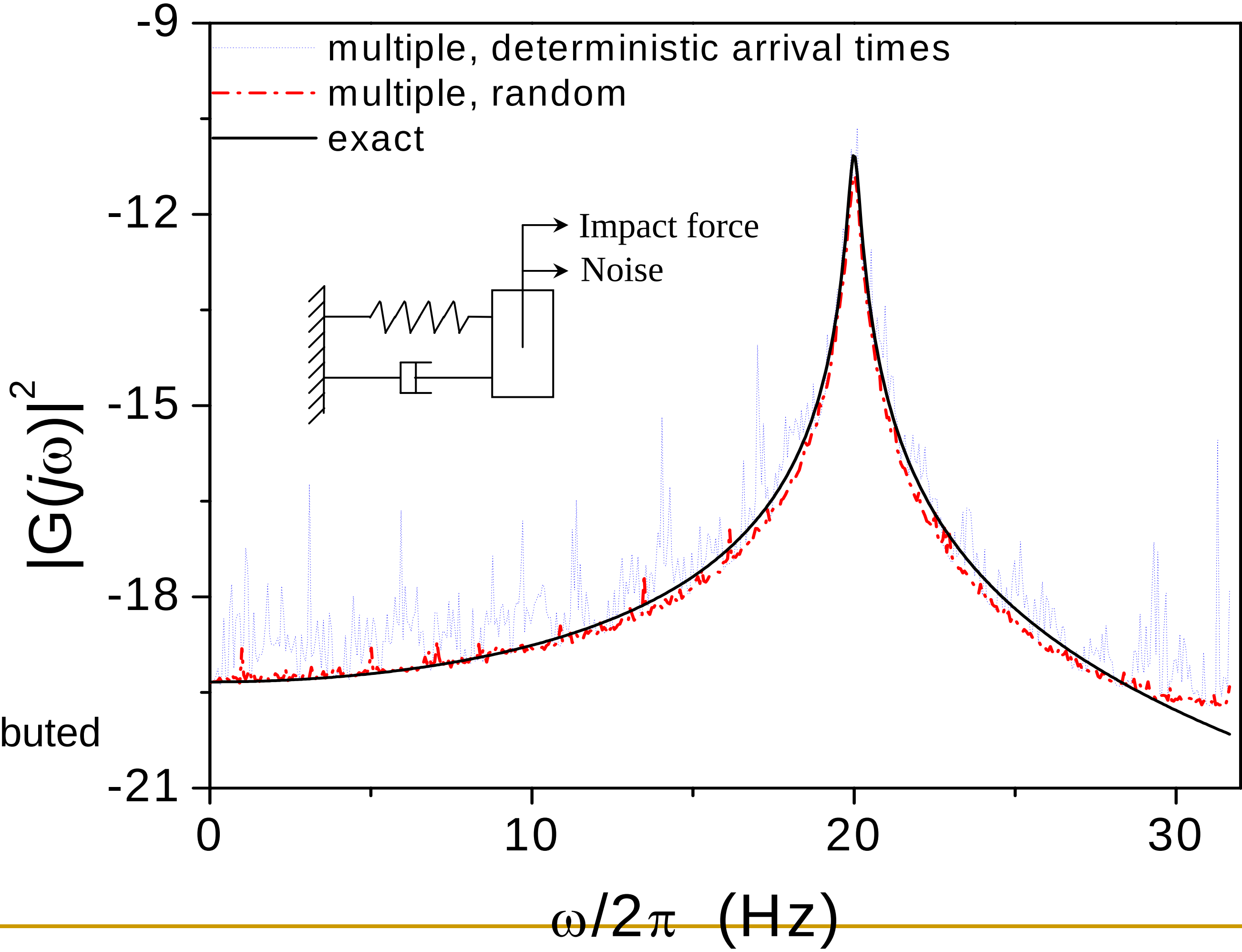
  T=11.39 s

- Arrival times

  t_i =0.2424*i* s (Deterministic)

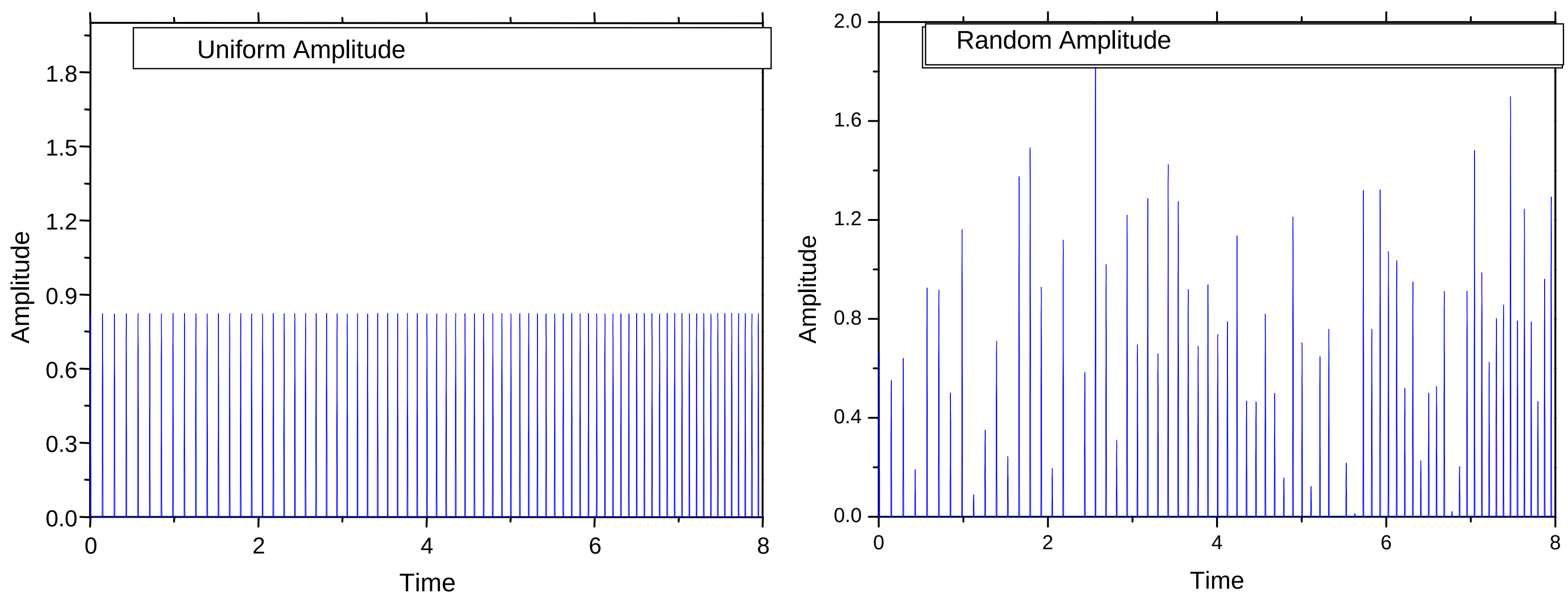
  Where i=1, 2,…, 66

  t_i =Uniformly distributed random variables (Random)



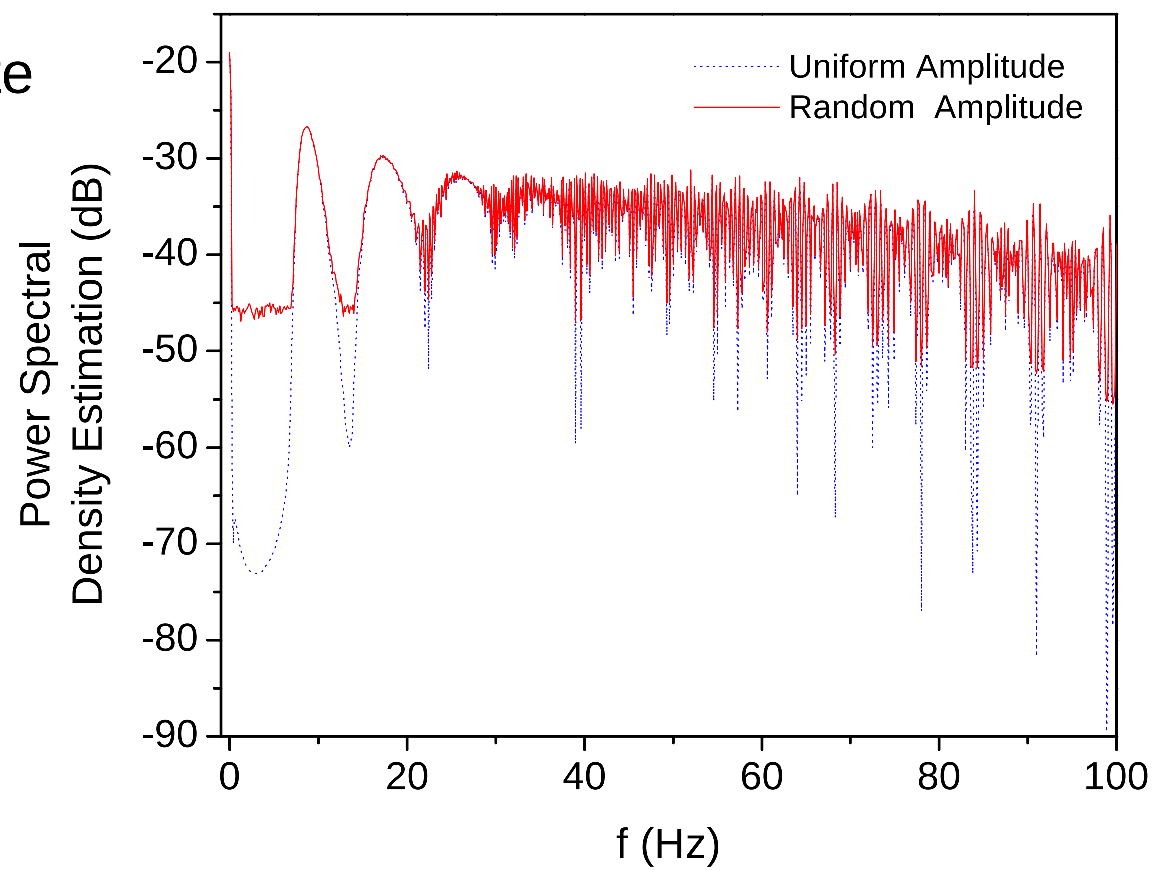
Random Impact Series with a Controlled Spectrum (RISCS)

- RISCS can concentrate the energy to a desired frequency range. For example, if one wants to excite natural frequencies between 7-13 Hz. The frequency of impacts can gradually increase from 7 Hz at t=0 s to 13 Hz at t=8 s.



Random Impact Series with a Controlled Spectrum (RISCS) (cont.)

- RISCS Can Concentrate the Input Energy in a Desired Frequency Range and the Randomness of the Amplitude Can Greatly Increase the Energy Levels of the Valleys in the Spectrum



Conclusions

- Novel stochastic models were developed to describe a random impact series in modal testing. They can be used to develop random impact devices, and to improve the measured frequency response functions.

- The analytical solutions were validated numerically.

- The random impact hammer test can yield more accurate test results for the damping ratios than the single impact hammer test.

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